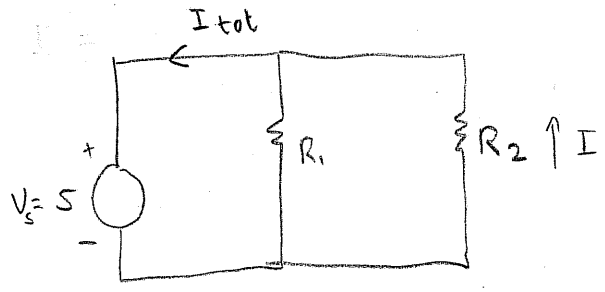
Problem 2.6

a) $I = -\frac{V}{R} = -5/2700 = -1.85\,mA$

b)

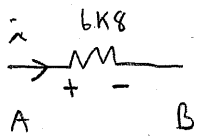


$I = -V_s/R_2 = -1.85\,mA$

(R_1 has no influence on the current thro' R_2)

c) $I = \frac{3-7}{3300} = -1.08\,mA$

2.7.

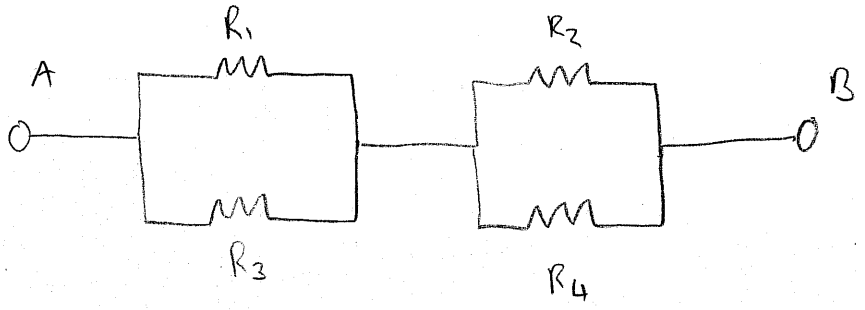


$i = 3\,mA$

A is at higher potential

$V = iR = 3\times10^{-3} \times 6800 = 20.4\,V$

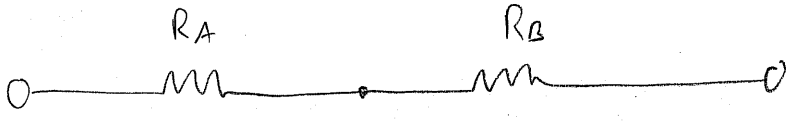
2.11



$R_A = R_1 \| R_3 = \frac{R_1 R_3}{R_1 + R_3}$

$R_B = R_2 \| R_4 = \frac{R_2 R_4}{R_2 + R_4}$

↓



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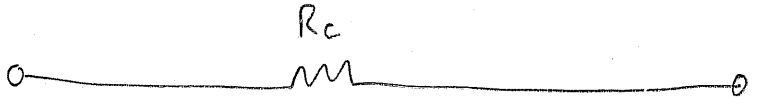


$$R_C = R_A + R_B = \frac{R_1 R_3}{R_1 + R_3} + \frac{R_2 R_4}{R_2 + R_4}$$

$$= \frac{5\times3.3}{5+3.3} + \frac{4.7\times8.2}{4.7+8.2} = 4.98\,k\Omega$$

2.28

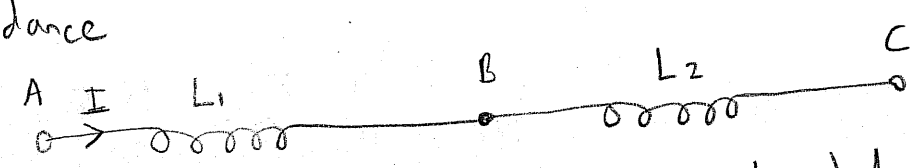
a) $$V_1 = 9 \times \frac{(330+470)}{330+470+270} = 9 \cdot \frac{800}{1070} = 6.73\,V$$

b) $$V_1 = V_0 \times \frac{R_2 \| R_3}{R_2 \| R_3 + R_1}$$ Now, $$R_2 \| R_3 = \frac{R_2 R_3}{R_2 + R_3}$$

$$= V_0 \frac{\frac{R_2 R_3}{R_2+R_3}}{\frac{R_2 R_3}{R_2+R_3} + R_1} = \frac{V_0 R_2 R_3}{R_2 R_3 + R_1 (R_2 + R_3)}$$

Impedance

5.2



Prove: Inductance between A and C, denoted L_{AC} equals $L_1 + L_2$.

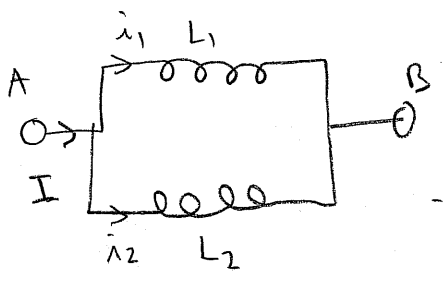
Proof: Define current I from A to C.

$$V_{AB} = L_1 \frac{dI}{dt} \qquad V_{BC} = L_2 \frac{dI}{dt}$$

$$V_{AC} = V_{AB} + V_{BC} = L_1 \frac{dI}{dt} + L_2 \frac{dI}{dt} = (L_1 + L_2) \frac{dI}{dt}$$

$$\therefore L_{AC} = L_1 + L_2 .$$

5.3



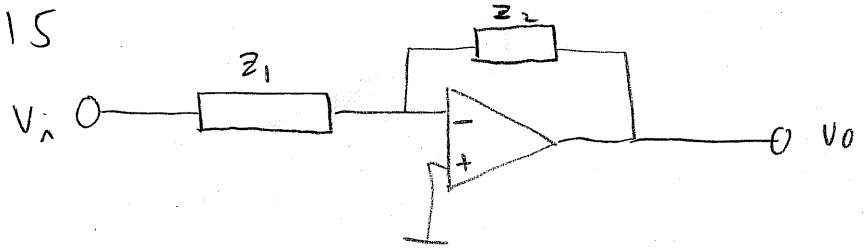
$$V_{AB} = L_1 \frac{di_1}{dt} = L_2 \frac{di_2}{dt} = L \frac{dI}{dt}$$

now $I = i_1 + i_2$ so $\frac{dI}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$

$$\therefore \frac{L_1 di_1}{dt} = L \left(\frac{di_1}{dt} + \frac{L_1}{L_2} \frac{di_1}{dt} \right)$$

$$\therefore \boxed{L = \frac{L_1 L_2}{L_2 + L_1}}$$

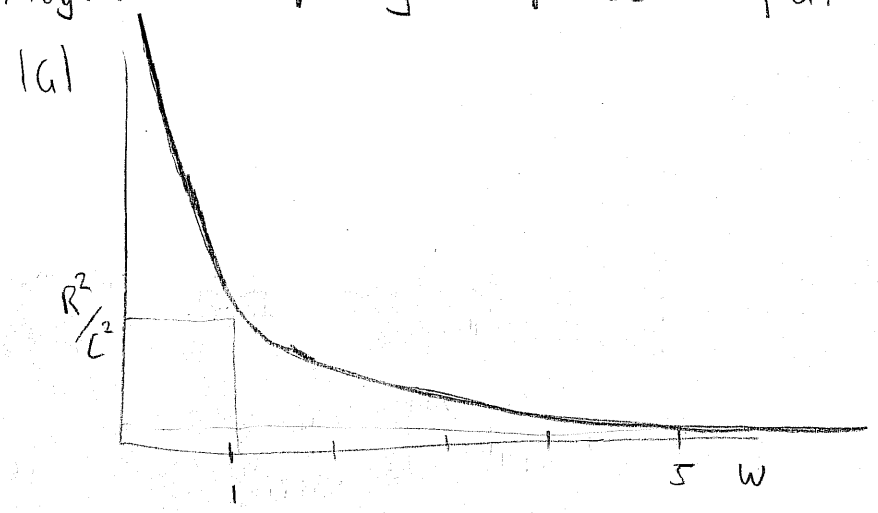
5.15



$Z_1 = R \quad Z_2 = \frac{1}{j\omega C}$ For inverting amp $G = \frac{V_o}{V_i} = -\frac{Z_2}{Z_1}$

So: $G = -\frac{1}{j\omega C} \Big/ R = -\frac{R}{j\omega C}$.

Magnitude frequency response: $|G| = \sqrt{G G^*} = \frac{-R}{j\omega C}\frac{R}{j\omega C} = \frac{R^2}{\omega^2 C^2}$

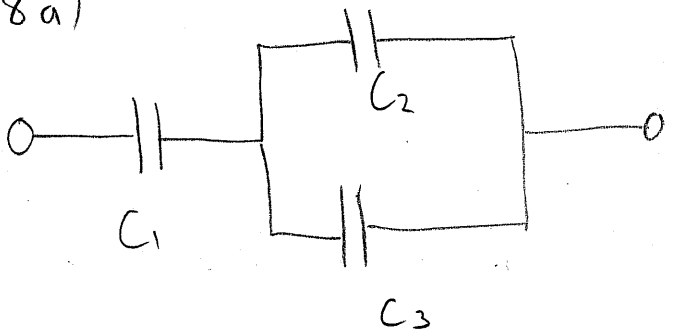


Why is the gain infinite at DC?

Remember, we are analyzing circuits in the steady-state: after all transients have decayed. In other words "for $t \to \infty$." The integrator keeps on integrating: the circuit never reaches steady-state, since the magnitude of V_o will increase monotonically for a DC input.

$\boxed{\text{AC circuit analysis assumes a steady state is reached}}$

5.8a)

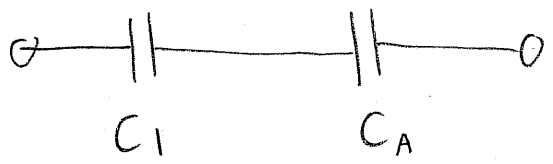


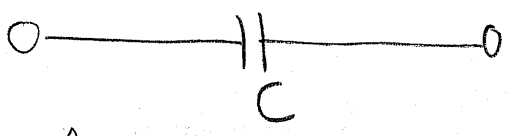
$$C_A = C_2 + C_3$$

$$C = \frac{C_1 C_A}{C_1 + C_A} = \frac{C_1 (C_2 + C_3)}{C_1 + C_2 + C_3}$$

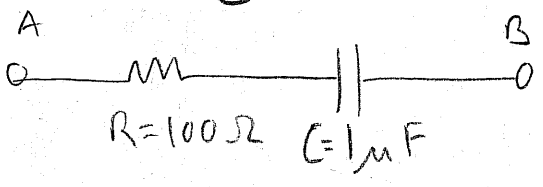
$$= \frac{15nF\,(23)}{15 + 23}$$

$$= 9.07nF$$





7.7.



$$X_c = -\frac{1}{wC}$$

$$Z_{AB} = R + \frac{1}{jwC}$$

$$Re(Z_{AB}) = R_{AB} = R = 100\Omega$$

$$Im(Z_{AB}) = X_{AB} = -\frac{1}{wC} = -\frac{1}{1000 \times 1 \times 10^{-6}} = -1000\Omega$$

7.9 a) $$Z_{AB} = R + \frac{1}{jwC} = R - \frac{j}{wC} = 100 - \frac{j}{w \times 1 \times 10^{-6}}$$

$$= 100 - j10^6/w$$

b) $$Z_{AB} = \frac{Z_C Z_R}{Z_C + Z_R} = \frac{\frac{R}{jwC}}{1/jwC + R} = \frac{R}{1 + jwCR} \cdot \frac{(1 - jwCR)}{(1 - jwCR)}$$

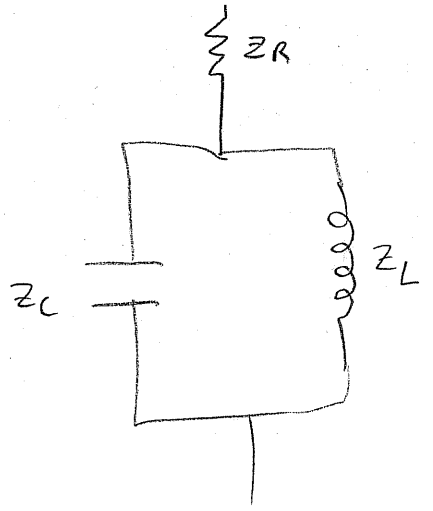
$$= \frac{R - jwCR^2}{1 + w^2R^2C^2}$$

7.9 c

$$Z_{AB} = Z_R + Z_C \| Z_L$$

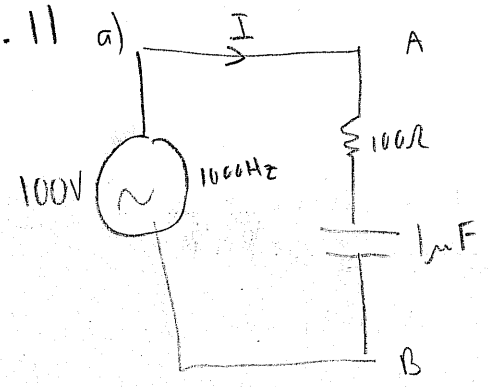
$$= R + \frac{Z_C Z_L}{Z_C + Z_L}$$

$$= R + \frac{\frac{1}{j\omega C} j\omega L}{\frac{1}{j\omega C} + j\omega L}$$



$$= R + \frac{j\omega L}{1 + j\omega C j\omega L} = R + \frac{j\omega L}{1 - \omega^2 LC}$$

7.11 a)



$$Z = 100 - j\frac{10^6}{\omega}$$

$$|Z| = \sqrt{\left(100 - j\frac{10^6}{\omega}\right)\left(100 + j\frac{10^6}{\omega}\right)}$$

$$= \sqrt{100^2 + \frac{10^{12}}{\omega^2}}$$

$$= 187.96\,\Omega$$

$$|I| = \frac{100}{187.96}$$

$$= 0.532\,A$$

b)

$$|I| = \frac{100}{|Z|}$$

$$= 394.8\,mA$$

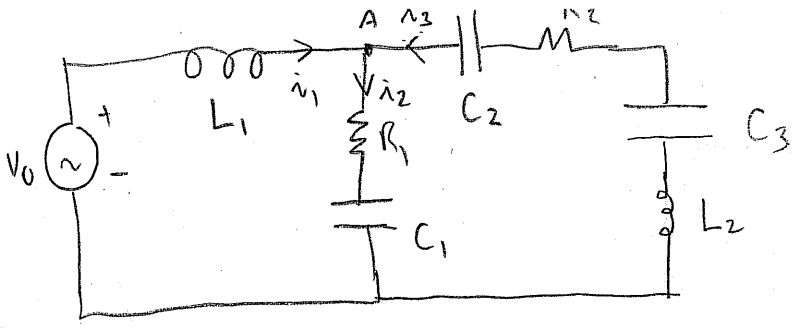
$$|Z| = \left[\frac{1}{\omega^2 R^2 C^2}\right]\sqrt{\left[R - j\omega C R^2\right]\left[R + j\omega C R^2\right]}$$

$$= \frac{1}{\omega^2 R^2 C^2}\sqrt{R^2 + \omega^2 C^2 R^4}$$

$$= 253\,\Omega$$

c) $|I| = 0.1299\,A$

7.19



Unknowns: V_A, i_1, i_2, i_3 — Need 4 eqns.

$$V_A - V_0 = i_1 Z_{L_1} \quad (1) \qquad i_1 = i_2 - i_3 \quad (2)$$

$$i_2 = \frac{V_A}{R_1 + Z_{C_1}} \quad (3) \qquad i_3 = \frac{V_A}{R_2 + Z_{L_2} + Z_{C_2} + Z_{C_3}} \quad (4)$$

(2):

$$i_1 = i_2 - i_3 = V_A \left[\underbrace{\frac{1}{R_1 + Z_{C_1}}}_{1/Z_A} - \underbrace{\frac{1}{R_2 + Z_{L_2} + Z_{C_2} + Z_{C_3}}}_{1/Z_B} \right]$$

$$1/Z_A = \frac{1}{R_1 + 1/j\omega C_1} = \frac{j\omega C_1}{R_1 j\omega C_1 + 1}$$

$$1/Z_B = \frac{1}{R_2 + j\omega L_2 + 1/j\omega C_2 + 1/j\omega C_3} = \frac{j\omega C_2 j\omega C_3}{R_2 j\omega C_2 j\omega C_3 + j\omega L_2 j\omega C_2 j\omega C_3 + j\omega C_3 + j\omega C_2}$$

$$= \frac{-\omega^2 C_2 C_3}{-\omega^2 R_2 C_2 C_3 - j\omega^3 L_2 C_2 C_3 + j\omega C_3 + j\omega C_2}$$

(1):

$$V_A - V_0 = V_A \left[\frac{1}{Z_A} - \frac{1}{Z_B} \right] Z_{L_1}$$

$$\boxed{V_A = \frac{V_0}{\left[1 - \left[\frac{1}{Z_A} - \frac{1}{Z_B} \right] Z_{L_1} \right]}}$$

where $Z_L = j\omega L_1$