EE C145B / BioE C165 Spring 2004: Midterm

READ THIS BEFORE YOU BEGIN:

Questions 1,2 and 3 are compulsory. You must choose to answer six questions from among 4-12. You may answer bonus questions 13-15 if you wish. If you answer more than six questions from among 4-12, only the first six of these will be graded unless you delete the answers to those you do not wish to be graded. Questions 8 and 11 also contain bonus points. You must include either or both of these among the six elective questions if you wish to score these bonus points.

Questions marked with an asterisk are more difficult. It may be prudent to answer them towards the end of the examination period.

READ THROUGH ALL QUESTIONS BEFORE YOU BEGIN.

Total points: 210 + 55 bonus.

Question 1 (46) COMPULSORY

Match the following to elements of the list below. More than one match is possible for each. You must list all the matches (in ascending numerical order) to get credit for each. Ask yourself “is this match always true?”.

1. 2D convolution matrix ________________
2. 2D Hamming window (in space) ________________
3. histogram equalization ________________
4. left singular vectors or matrix containing them ________________
5. SVD ________________
6. Radon transform ________________
7. median filter ________________
8. right singular vectors or matrix containing them ________________
9. singular values or diagonal matrix containing them ________________
10. DFT ________________
11. DSFT ________________
12. 1st derivative edge detector ________________
13. FT ________________
14. ideal ramp filter (analogue) ________________

15. ideal reconstruction filter ______________________

16. convolution ______________________

17. $h(x, y) = 3x^2$ ______________________

18. aliasing ______________________

19. Laplacian edge detector (discrete) ______________________

20. 2D discrete Radon transform matrix ______________________

21. Butterworth filter ______________________

22. Windowing ______________________

23. jinc function ______________________

List of possible matches:

1. Convolution matrix
2. non-linear
3. decreases spectral resolution
4. decreases spatial resolution
5. space domain
6. frequency domain
7. span range
8. span nullspace
9. non-negative
10. positive
11. attenuates edge effects
12. negative
13. non-positive
14. periodic in space domain
15. periodic in frequency domain
16. discrete frequency axis
17. discrete space axis
18. high pass
19. low pass
20. maximally flat in passband
21. lowest phase distortion
22. all pass

23. mutually normal
24. linear
25. conserves energy / mass
26. block Toeplitz
27. contains all radial frequencies up to cutoff
28. Toeplitz
29. separable
30. zero when applied to constant gray level
31. zero when applied to gray level ramp
32. mutually orthogonal
33. unitary
34. full rank
35. digital filter

Question 2 (20) COMPULSORY

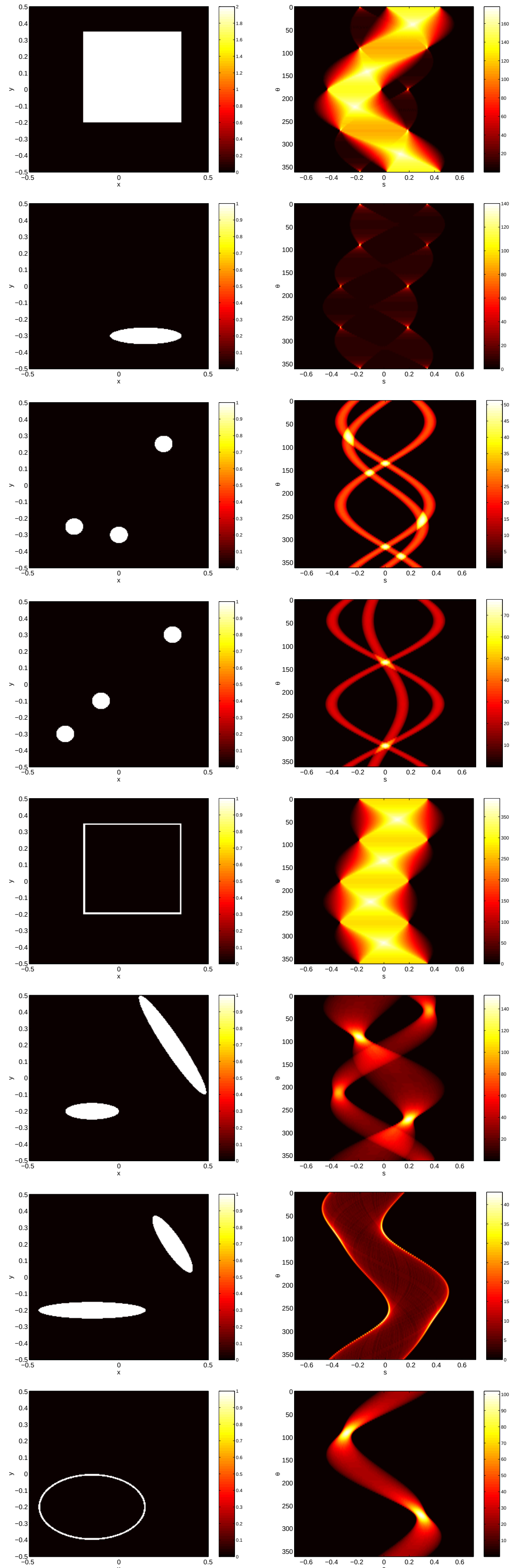
You are given the following matrix that represents the gray levels of an image:

$$\begin{bmatrix} 0.8 & 0.8 & 0.4 & 0.8 \\ 0.7 & 0.1 & 0.3 & 1.0 \\ 0.5 & 0.6 & 0.9 & 1.0 \\ 0.6 & 0.1 & 0.0 & 0.8 \end{bmatrix}$$

Equalize the histogram of this image. Show all steps. Use five histogram bins spanning the range $[0, 10]$. Map values to the lowest gray level in each bin.

Question 3 (24) COMPULSORY

Match the elements in the left column to those in the right that could possibly be their Radon transforms. Enter the letter identifying the match on the line to the left of each row.



a

b

c

d

e

f

g

h

Question 4 (20)

Show mathematically and graphically, starting with an arbitrary continuous space signal $f(x)$ why its DFT is periodic and why the DFT assumes the samples of this signal are periodic. Assume signal is sampled without aliasing at $u_s = X$ samples per distance unit. Hint: apply multiplication-convolution FT property, and use periodic combs of impulses to represent sampling.

Question 5 (20)

1. Explain the difference between image enhancement and restoration.

2. You are working on a project to count stars visible in the sky. Hundreds of images captured by your telescope one night are blurred and seem to also to have poor contrast. Before you move the telescope after the last image is acquired, you refocus it and store the image. What is the most effective way to deblur the image given the available data?

3. The image is now crisp, but the contrast is still poor. You blame the lights of a nearby sports field for brightening the background when your boss threatens you with dismissal. What can you do to make amends, given the object is not to make pretty pictures of the sky, but to count stars?

Question 6 (20)

1. Explain, with reference to an equation from your infosheet, how the SVD can be used to compress images.

2. You are given the task of compressing all of the aerial survey photographs of the United States so that these can be conveniently accessed over the Internet. You decide to use the SVD to do this to save bandwidth. Each photo has been digitized as an 8000×8000 grayscale pixel image of 10 bit words. For the unzoomed 1:50,000 scale view, you find that 150 singular vectors are sufficient. Calculate the fraction of memory saved.

Question 7 (20)

Use the projection slice theorem to prove the rotation property of the Radon transform. Hint: This is a two to three line proof.

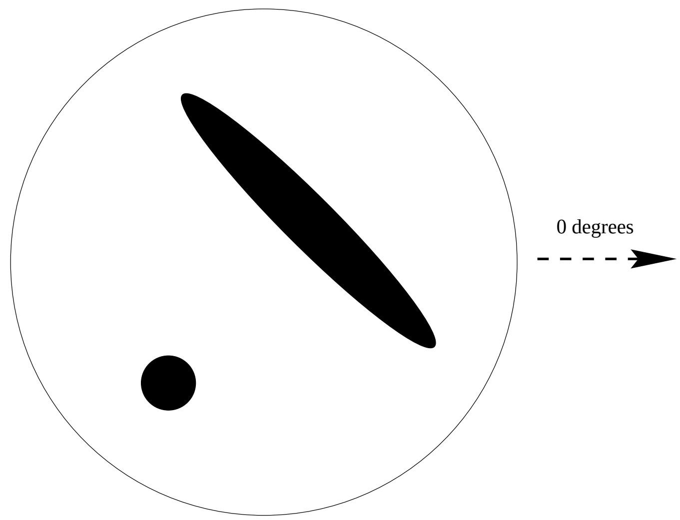
Question 8 ($20 + 10$ bonus)

1. Argue, using the projection theorem and a Fourier transform pair in your info sheet, that the Radon transform of a 2D Gaussian must be a 1D Gaussian.

2. For what other function must this also be true?

Question 9 (20)

1. Draw the sinogram of the objects in the distribution below. The parameter θ corresponds to the direction of $\boldsymbol{\theta}$ as defined in the Radon transform. Label the axes.



Question 10 (20)

The following matrix contains a cropped area of a larger image:

$$\begin{bmatrix} 2 & 5 & 3 & 0 & 0 \\ 8 & 7 & 1 & 8 & 2 \\ 5 & 4 & 3 & 0 & 5 \\ 1 & 2 & 9 & 11 & 2 \\ 6 & 5 & 3 & 8 & 4 \\ 1 & 0 & 6 & 7 & 2 \end{bmatrix}$$

Consider the element at row 3 and column 3 of this matrix.

1. What would a 5×5 median filter replace this value with? Specify your conventions.
2. What would a 3×3 max filter replace this value with?

Question 11 ($20 + 10$ bonus)

Prove that the values of $F(u, v)$ along the radial line $v = mu + c$ where $c = 0$ in 2D Fourier space correspond to the 1D Fourier transform of the Radon projection of $\mathcal{F}_1^{-1}\left\{F(u, v)\right\}$ for a particular value of $\boldsymbol{\theta}$. Express $\boldsymbol{\theta}$ in terms of m. Hint: This is simply the proof of the projection theorem run through backwards.

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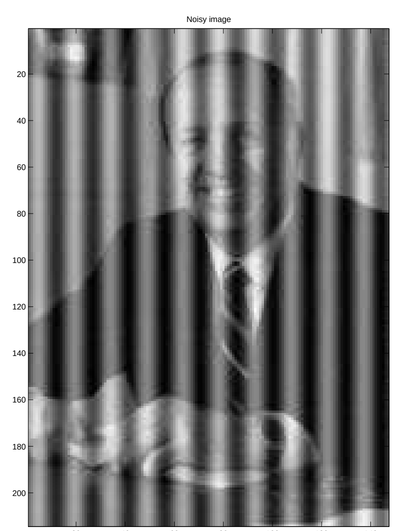
Question 12 (20)

Show diagramatically how to remove the artifact in the image below.



1. Using a 1D digital filter:

2. Using an analog method:

Question 13 (10 bonus)

We have seen in Problem Set II that any image can be viewed as an interference pattern of different 2D sinusoidal functions. You take a 1024×768 picture in the Golden Gate Park that you want to use for your computer desktop background. Your default display has a resolution of 1600×1200 pixels. You use photoshop to modify your image so that you have:

Image size = 16×12 cm
Resolution = 100 pixels/cm
What is the frequency of the highest frequency sinusoid that may be present in your image?

Question 14 (15 bonus)

* Explain mathematically how you could calculate the SVD of a matrix $\mathbf{F}$ given that you know how to find the eigenvectors and eigenvalues of a matrix. Give two reasons why this is not such a good way of finding the SVD.

Question 15 (20 bonus)

* In class, we derived the pseudoinverse by formulating the sum of squared residuals:

$$C = \sum_{m=1}^{M} r_m^2 = \sum_{m=1}^{M} \left[\sum_{n=1}^{N} f_n^m(x_m)\,\theta_n - y_m \right]^2$$

and minimizing it by taking its derivative with respect to each of the parameters θ_n, setting these equations equal to zero, and solving for $\boldsymbol{\theta}$. It is much easier to derive the pseudoinverse from the vector formulation. Show the matrix equation that gives the vector that minimizes C. Hint:

$$\frac{d}{d\mathbf{x}}\,\mathbf{Q}\,\mathbf{x} = \mathbf{Q}^T$$

$$\frac{d}{d\mathbf{x}}\,\mathbf{q}\,\mathbf{x} = \mathbf{q}^T$$

$$\frac{d}{d\mathbf{x}}\,\mathbf{x}^T\,\mathbf{Q}\,\mathbf{x} = 2\,\mathbf{Q}\,\mathbf{x}, \text{for } \mathbf{Q} = \mathbf{Q}^T$$

$$\frac{d}{d\mathbf{x}}\,\mathbf{x}^T\,\mathbf{Q} = \mathbf{Q}$$

$$\frac{d}{d\mathbf{x}}\,\mathbf{x}^T = \mathbf{I}$$