EECS C145B / BioE C165 Spring 2003: Problem Set III (Practice midterm) Due March 18 2003

For this problem set, each student must submit their own solutions.

Questions marked with an asterisk are more difficult. It may be prudent to answer them towards the end of the examination period.

Total points: 370 + 55 bonus.

Question 1 ($(10 + 10) + 10$ bonus)

1. Calculate by hand the 3rd term of the DFT of the function

$$f[n] = \{1, 1, 1, 1, 0, 0, 0, 0\}.$$

   Denote this term as $F[2]$.

2. How does it compare to the third term ($G[2]$) of

$$g[n] = \{0, 0, 1, 1, 1, 1, 0, 0\}?$$

3. * Considering the magnitude spectra $|F[k]|$ and $|G[k]|$ of $f[n]$ and $g[n]$ respectively, why was there no need to calculate the result for part (2)? What Fourier transform property is demonstrated here?

Question 2 $(10+10)$

Convolve the following images:

1.

$$f[m,n] = 5\,\delta[m-1,n] - 3\,\delta[m+1,n+1]$$

$$g[m,n] = 2\delta[m-1,n]$$

2. *

$$f[m,n] = 5\,\delta[m-1,n] - 3\,\delta[m+1,n+1] - 3$$

$$g[m,n] = 2\,\delta[m-1,n]$$

Draw the results, labeling each Kroneker delta with its amplitude.

Question 3 $(2 \times 8 = 16)$

Are the following systems linear, spatially invariant, both or neither? Assume g is the system output and f is the system input.

1. $g(x, y, z) = z$
2. $g(x, y, z) = 1$
3. $g(x, y) = f(3x - 2, y - 6) + 3f(x + 3, y - 3)$
4. $g(x) = \sin(x)$
5. $\mathbf{g} = \mathbf{F}\,\mathbf{f}$
6. $\mathbf{g} = \mathbf{F}^{+}\mathbf{f}$
7. * $g(x, y, z) = 5\,g(x, y, z)$
8. * $g(x, y) = f(x - y, y - 6) + 3f(x + 3, y - 3)$

Question 4 $(20+4)$

The 8-element vector

$$\left[\; 3, \quad -4\jmath, \quad 0, \quad 1+\jmath, \quad 1, \quad 1-\jmath, \quad 0, \quad 4\jmath \;\right] \times 8 \tag{1}$$

corresponds to the DFT of a signal.

1. Draw the sinusoidal components of this signal on separate but identical axes.

2. What operation must be performed on these five signals to yield the original signal?

Question 5 (15)

Explain, with the aid of a diagram if necessary, the reason why a square image with square pixels has its greatest sampling rate along the diagonal.

Question 6 (10)

Explain the difference between image enhancement and restoration.

Question 7 ($(10 + 10) + 10$ bonus)

1. Describe what windowing is and why it is sometimes desirable. Also describe its drawbacks, especially as they relate to imaging.

2. Without regard for absolute amplitude scale, draw the FT of the of the continuous Hamming function:

$$w(t) = \{ \ (0.54 - 0.46\cos(2\pi t/T))$$

3. * With the convolution theorem in mind, describe what impact multiplication by a Hamming window in 1D space has on a single spectral peak in the Fourier domain.

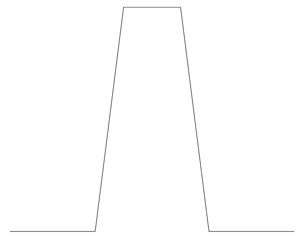
Question 8 $(15 + 5 + 10)$

You are required to reconstruct an image using backprojection of filtered projections. The noise-free projections have 8 bins. Filtering is performed by multiplying, element by element, the DFT of each projection with a ramp filter. You wish to maximize the resolution of the reconstructed image.

1. Assuming there is no noise present in the projections, draw the ramp filter used to effect the ramp filtering. Remember to label the axes of your drawing, giving the frequency at each of the 8 points. Do not worry about absolute amplitude scaling. Recall that for the DFT, zero frequency occurs at index 0.

2. In the presence of noise, how would you modify this filter?

3. Show how the following projection is processed before backprojection:



$\downarrow$

Question 9 (15)

Show diagramatically, how 2D ramp filtering of a backprojection image may be performed using analog components only.

Question 10 $(15 + 10)$

You take a picture of a crime in progress from a moving car traveling at a constant speed of 20 m/s. You take the photo looking out of the passenger seat window. You check the shutter speed setting on the camera, and it is set to keep the shutter open for one hundreth of a second.

1. *Draw the 2D blurring kernel and label the x-axis. (Note, if you can't do this part of the question, assume you know the kernel and do part 2).

2. Describe the steps you would take to deblur the scene.

Question 11 (15)

The following matrix contains a cropped area of a larger image:

$$\begin{bmatrix} 5 & 4 & 3 & 0 \\ 1 & 2 & 9 & 11 \\ 6 & 5 & 3 & 8 \\ 1 & 0 & 6 & 7 \end{bmatrix}$$

Consider the element at row 2 and column 3 of this matrix. What would a 3×3 median filter replace this value with?

Question 12 (15)

The following is the kernel of a Gaussian low-pass filter. Convert it to a Gaussian high-pass filter and write out the matrix.

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} /16$$

Question 13 (20)

Prove the 2D projection slice theorem (in vector or scalar form).

Question 14 (20)

Derive the formula from which the backprojection/filter tomographic reconstruction algorithms follow using the inverse FT and the projection slice theorem (in vector or scalar form).

Question 15 $(10 + 10)$

An image has the pixel values:

$$0, 1, 1, 2, 10, 10, 5, 3, 2, 2, 4, 9, 5, 5, 5, 4, 0, 0$$

Draw the four-bin image histogram and the cumulative histogram. Label the bin centers.

Question 16 (10)

Organize the vector given in the previous problem into a 6×3 image. Threshold this image using an upper threshold of 5 and a lower threshold of 2.

Question 17 (20)

You have at your disposal a 1D DFT engine and some backprojection code. Describe step-by-step, or in diagram form, two ways of reconstructing an image from projections using what you have.

Question 18 $(15 + 10)$

A rectangular block of lead is 2 cm long and 3 cm wide. Its center is positioned at the center of rotation of an x-ray CT camera.

1. Approximately draw the parallel beam projections for $\boldsymbol{\theta}$ oriented at $0, 45$ and 90 degrees.

2. If the attenuation coefficient of lead is μ per meter, find an expression for the amplitude of the central bin of the 45 degree projection.

Question 19 (20)

Show graphically the steps needed to equalize the histogram of an image.

Question 20 ($10 + 10$ bonus)

Given the SVD:

$$
\begin{aligned}
\mathbf{F} &= \begin{bmatrix} 1 & -2 \\ 2 & 2 \\ 2 & -4 \end{bmatrix} \\
&= \mathbf{USV}^T \\
&= \begin{bmatrix} 0.4342 & 0.1072 \\ -0.2398 & 0.9708 \\ 0.8683 & 0.2145 \end{bmatrix} \begin{bmatrix} 5.1093 & 0 \\ 0 & 2.6259 \end{bmatrix} \begin{bmatrix} 0.3310 & -0.9436 \\ 0.9436 & 0.3310 \end{bmatrix}
\end{aligned}
$$

1. Determine the dimension of the range and nullspace of $\mathbf{F}$.

2. * Is every vector in the 3D real space in the range of $\mathbf{F}$? Explain why using a geometric argument.

Question 21 (25 bonus)

* The eigenvalue decomposition of a matrix $\mathbf{X}$ can be written as:

$$\mathbf{XZ} = \mathbf{Z\Lambda}$$

where $\mathbf{Z}$ contains as columns the eigenvectors of $\mathbf{X}$, and $\mathbf{\Lambda}$ is a diagonal matrix containing the eigenvalues of $\mathbf{X}$. The eigenvalues are sorted in descending order along the diagonal of $\mathbf{\Lambda}$. The corresponding eigenvectors are arranged in $\mathbf{Z}$ in the same order.

The singular value decomposition of $\mathbf{F}$ is given by:

$$\mathbf{F} = \mathbf{USV}^T$$

Assuming that the eigenvectors of $\mathbf{F}^T\mathbf{F}$ are the same at the right singular vectors of $\mathbf{F}$, show that the eigenvalues of $\mathbf{F}^T\mathbf{F}$ are the squares of the corresponding singular values of $\mathbf{F}$.